Modeling of the Ski-Snow Contact for a Carved Turn

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Abstract. Carved turns with alpine skis are investigated. During the movement of a ski, snow is loaded and unloaded. Compacted snow is not elastic, i.e. deformations remain. Such effects are modeled by a hypoplastic constitutive equation. During a turn the shovel digs into the snow and the tail maintains nearly the same penetration depth as the part under maximum load. This results in a higher resistance against shearing for the afterbody of the ski. In the present work we investigated the benefits of the hypoplastic against the elastic force-penetration relationship. Simulation results for a sledge on two skis are compared to experimental track data.

1 Introduction

Skiing experts define a perfect turn as carving with no skidding. Therefore we studied carved turns with alpine skis and in particular the contact problem between ski and snow. To avoid influences of the skier's motions, the movement of a rigid sledge on two skis was investigated. A common approach is to model the penetration force by non-linear springs. However, snow is not elastic. Deformations of the snow remain, leaving tracks. Such effects can be modeled by a hypoplastic force-penetration relationship (e.g. Fellin 2000). During a turn, the ski is bent in a way that the loaded edge has the shape of a circular arc. The shovel digs into the snow and the tail maintains nearly the same penetration depth as the part under maximum load. This results in a higher resistance against shearing in the afterbody of the ski.

There exist a number of publications dealing with skiing in general, the turn radius and the ski-snow contact. Basic textbooks on skiing have been written by Howe (1983) as well as by Lind and Sanders (1997). The turn radius and its implication to load were discussed in Mössner et al (1997). One of the first attempts to solve the ski-snow contact problem was by Renshaw and Mote (1991). They used the force-penetration relationship found by Lieu and Mote (1984) for ice. Kaps et al (2001) studied the quasi-static pressure distribution along the running surface in a carved turn. Tada and Hirano (1999) used an analogy to water-jets and shearing principles from cutting theory. Nordt et al (1999) used linear spring forces for penetration and cutting theory for shearing forces. They were the first to use a realistic implementation of the ski geometry. Bruck et al (2003) used measurement data to fix the springs for the penetration force and to fix the shearing force. Finally Federolf (2005) implemented an FE-ski and used own measurement data to assign the snow properties.

For this work we started from the implementation of Bruck et al (2003). The main aim of this study was to investigate the benefits of an improved implementation of the ski-snow contact. To get smooth input data for the force calculations routines we introduced a smooth surface, referred as the running surface of the ski, between the segments of the mechanical ski model and the snow. All forces were calculated with respect to this surface. The force-penetration relationship was improved from pure elastic to hypoplastic response. Whenever possible, improved knowledge of snow properties was used. Differences in the behavior will be shown and benefits outlined. Simulation results will be compared to experimental track data.

2 Method

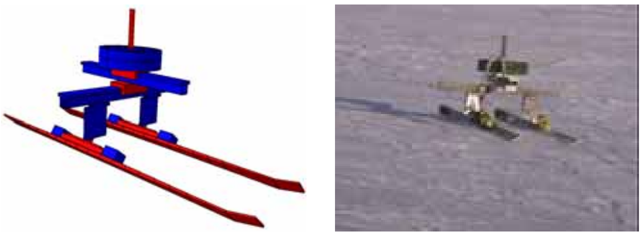

Fig. 1. The sledge on skis in the LMS Virtual.Lab software and in the validation experiment.

2.1 The Sledge and the Skis

A rigid sledge on two skis was constructed (Fig 1, right). The sledge had two feet with ski-boot soles which were fitted to the bindings of the skis. The angle of these feet could be adjusted to a given value. This resulted in a fixed but user prescribed value for the edging angle of the skis. On the top of the sledge a pole was mounted to hold additional weights. The position of this pole could be shifted side to side as well as front to rear so that the center of mass could be varied.

In the multi body system (MBS) software LMS Virtual.Lab the sledge was implemented as one complex rigid body (Fig 1, left). Each ski consisted of 18 rigid segments and a shovel segment. The segments were linked to each other by revolute joints that allowed rotation around the transversal (bending) and the longitudinal (torsion) axes. The spring and damping constants for these joints were taken from experiments on real skis. The boot sole was connected to the bindings by spherical joints. The binding-ski connections were a bracket joint for the toe piece and a translational joint for the heel piece.

2.2 Ski-Snow Contact

The MBS implementation of the skis consists of rigid cuboids that are joined along the bending line. Via the bottom surfaces of the cuboids the reaction forces of the snow have to be applied. Since the rectangles are joined along the bending line only, the contact surface is not smooth and the edge is discontinuous. Therefore we attach a differentiable surface to the bottoms of the cuboids. The center line of this surface represents the bending line of the ski. It is given by a piecewise cubic Bezier curve which coincides with the centers and the directions of the centerlines of the base rectangles of the ski segments. In cross direction the surface is given by straight lines of length *w/2*, with *w* the ski width. This results in a G^1 smooth surface which is defined as the running surface of the ski. Every point of the bottom rectangles of the MBS model corresponds uniquely to one point of the running surface. Forces are calculated with respect to this surface and are supplied to the MBS model via the corresponding point of the cuboid ski segment.

Only ski-snow contact forces and the gravitational force are assumed to interact with the sledge. For ski-snow contact forces, the three types of forces considered are: 1) the penetration resistance force normal to the snow surface, 2) the shear force transversal to the ski movement, and 3) the friction force in tangential direction.

Penetration Force. For the reaction force normal to the snow surface we study two models: the elastic, and as improvement, the hypoplastic force-penetration relationship. For the force calculations each of the segments are divided further into 16 sub-segments. Consider one of these sub-segments having the length L, the edging angle β and the penetration depth $\overline{e}$ at the edge. Then this sub-segment displaces the volume

$$V = \frac{L\,\overline{e}^2}{2\tan\beta} \tag{1}$$

of snow. Let H be the snow hardness, resp., the compressibility of snow. Then, in the elastic case each point *(x, y)* of the running surface with snow-contact produces the snow pressure

$$p^e(x,y) = H\,e(x,y). \tag{2}$$

Integration over the contact surface A gives the reaction force for the elastic force-penetration relationship

$$F_p^e = \int_A p^e(x,y)\,dxdy = H\,V. \tag{3}$$

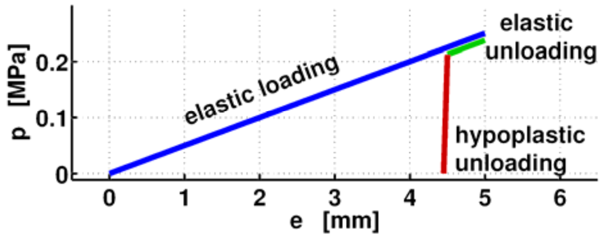

Fig. 2. Snow pressure for the hypoplastic force-penetration relationship.

$$p^h(x,y) = \begin{cases} H\,e(x,y), & 0 < e < e_{\max} \quad \textit{elastic loading} \\ H\,e(x,y), & e_2 < e < e_{\max} \quad \textit{elastic unloading} \\ H\,\dfrac{e(x,y)-e_1}{e_2-e_1}, & e_1 < e < e_2 \quad \textit{hypoplastic unloading} \\ 0, & 0 < e < e_1 \quad \textit{unloaded} \end{cases} \tag{4}$$

For the hypoplastic force-penetration relationship (Fig 2, Eq 4) one has to distinguish between loading and unloading of the snow. Whenever all frontal sub-segments have penetration depth less than the current sub-segment then snow is loaded. In this case the elastic branch of the force-penetration relationship is used. In the other case, when any frontal sub-segment has a penetration depth larger than the actual sub-segment, then the snow is unloaded. In this case the unloading branch of the force-penetration law has to be used. It begins with a small part of elastic unloading followed by a steep drop of the force response of the snow. When deciding if the case is snow loading or unloading, the decision is independent of the movement of the whole ski. However, for correctness one should check the penetration depth of the frontal sub-segments when passing the location of the current sub-segment. We omit this complication since penetration depth changes slowly with respect to travel velocity of the ski. Finally the reaction force for the hypoplastic force-penetration relationship is given by integration of the reaction pressure

$$F_p^h = \int_A p^h(x,y)\, dxdy. \tag{5}$$

Shear Force. The shear force is modeled in analogy to metal cutting theory (Shaw 1984). For metal cutting as well as for ice (Lieu and Mote 1984) it is known that shearing is almost independent of the shearing velocity. Given the ultimate shear pressure of snow p^*, which is related to the Young's modulus of snow, the reaction force for a sub-segment is given by

$$F_s = p^* e L. \tag{6}$$

We use a smooth crossing for lateral velocities between zero and the threshold velocity v^*.

Friction Force. Finally friction is modeled using Coulomb friction

$$F_f = \mu F_p. \tag{7}$$

2.3 Data Collection

For validation of the simulation model a sledge on skis was constructed (Fig 1) and field experiments were conducted in Stuben/Tyrol (Mössner et al 2001). Several runs of the sledge were performed on a planar slope of 13° inclination and the sledge's path was surveyed using a theodolite. The angle φ between fall-line and the tangent of the track was calculated from the track data. The initial velocity v_0 was selected to obtain the correct running time in the simulation. The track length s was calculated from the data points. The total run time T was taken from video recordings. The edging angle of the skis in the experiment was $\beta = 35°$. Snow hardness H and ultimate shear pressure p^* were measured several times during the runs of the sledge (Mössner et al 2003). Two runs of the sledge with two types of skis were used for the validation. The first ski (WC) was a giant slalom ski and the second (XT) a carver ski. The data for the runs were collected in Tab 1. Geometry and mass properties were collected for the skis and the sledge. Finally the skis bending and torsional stiffness were measured.

In addition unknown model parameters were fixed. From unpublished measurements we know that unloading shows negligible elastic response, i.e. the slope of the hypoplastic branch is very steep. On the other hand some elastic response at the start

ski	Date	time	φ	$\boldsymbol{v_0}$	S	$\boldsymbol{T}$	H	p^*	$\boldsymbol{r_S}$
WC	28th Feb	17^h10	44	0.8	62	9.6	0.67±0.08	150±24	32.0
XT	1st Mar	11^h35	72	2.0	38	8.0	0.12±0.05	57±8	12.5

Tab. 1. Data for the two runs of the sledge with the XT and the WC ski. φ[°] angle between fall-line and tangent of the track at start point, v_0 [m/s] initial velocity of the sledge, s [m] track length, T [s] running time, H [N/mm^3] snow hardness, p^* [kPa] ultimate snow pressure, r_S [m] nominal ski-radius. For the snow properties H and p^* the standard deviations of the measurements are given.

of unloading was introduced. We set $e_1 = 0.89\ e_{max}$ and $e_2 = 0.90\ e_{max}$. Knünz et al (2000) assessed the attack angle α of elite skiers performing optimal carved turns to be about 1° for the loaded ski. In the simulation model the selection of v^* allowed the ski some amount of skidding, which resulted in a small attack angle in the simulation. Because of this we set v^* = 0.03 m/s. According to measurements from Kaps et al (1996) we use $\mu = 0.07$ for the friction coefficient.

2.3 Validation of the Simulation

Using the model and the data from above we simulated the track of the loaded outside ski of the sledge. Let $(x_i, y_i),\ i = 1,...,n$ be the track points given by the measurement and $(X(t), Y(t)),\ 0 < t < T$ be the track of the 10th ski-segment, as obtained from the simulation. We calculated the distance

$$d = \max_i \min_{0<t<T} \sqrt{(X(t)-x_i)^2 + (Y(t)-y_i)^2}, \tag{8}$$

i.e. the maximum deviation of a data point from the simulated path. Additionally we checked the running time in the simulation against the running time in the experiment.

3 Results

Since snow conditions vary strongly on a slope we adjusted snow hardness H and ultimate shear pressure p^* in order to get optimal agreement of the track data with the track of the simulation. For the WC and XT skis, respectively, we selected $H = 0.40$ and 0.06 N/mm^3 and $p^*=140$ and 75 kPa. Fig 3 shows the track of the sledge for the WC (left) and the XT (right) skis. The maximum deviation (Eq 8) of the simulated track was for WC and XT 0.31and 0.17 m for the hypoplastic and 5.05 and 7.18 m for the elastic force-penetration relationship, respectively. The simulated track for the hypoplastic force-penetration relationship is in good agreement with the collected data points, whereas the simulated track for the elastic force-penetration relationship shows a constant drift off the data points.

4 Discussion

Two models for the ski-snow contact have been implemented and the simulation results were compared to real experiments. The hypoplastic force-penetration relationship proved to be superior to the elastic force-penetration relationship. The simpler elastic force-penetration relationship showed a steady drift off of the simulated track from the measured data points, whereas the complex hypoplastic force-penetration relationship showed convincing agreement with the data. In addition the penetration depth of the skis was more realistic for the hypoplastic than for the elastic force-penetration relationship. For the simulation with the hypoplastic model the tail of the ski penetrated deeper into the snow than for elastic force-penetration relationship. Deeper penetration allowed for better carving with less skidding, thus faster turn velocities were possible.

In the simulation the value of snow hardness H and ultimate shear pressure p^* had to be modified from the measurement data. One reason for this is that snow conditions vary considerably along the path of the sledge.

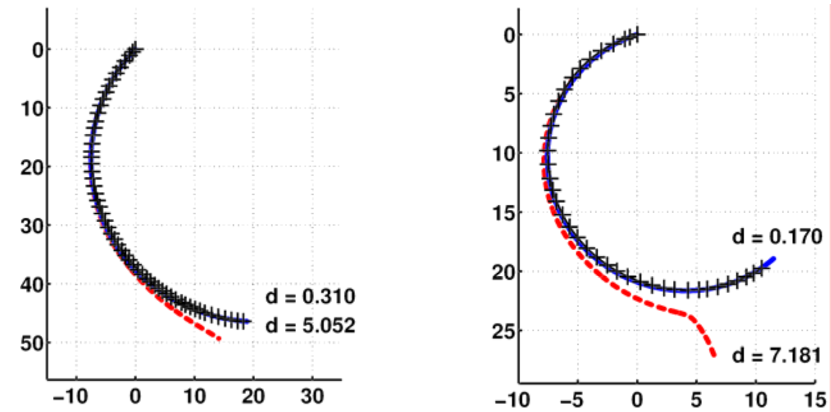

Fig. 3. Tracks of the sledge on skis for the WC (left) and XT (right). The coordinate axes are Cartesian coordinates [m] in the plane of movement (slope). The fall-line coincides with the *y*-axis. — track for the hypoplastic force-penetration relationship, - - track for the elastic force-penetration relationship, + data points of the measurement. Displayed are also the maximum deviations between simulation and data (Eq 8).

Because of the validity of our implementation the model can be used to study the influence of construction properties of the skis (bending/torsional stiffness, shape, ...), snow conditions (snow hardness, shearing properties) as well as actions of the skier (edging angle, front/back leaning, ...). First results on the influence of bending stiffness for various initial conditions are given by Heinrich et al (2006). One of our next aims will be the simulation of combinations of turns.

Acknowledgment

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